Toward improved ocean-atmosphere coupling algorithms

*Eric Blayo, Florian Lemarié,
Charles Pelletier and Sophie Thery*

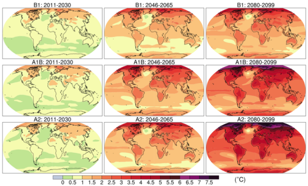
Univ. Grenoble Alpes and Inria, France

UNIVERSITÉ Grenoble Alpes

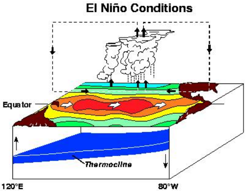
informatics mathematics Inria

Context

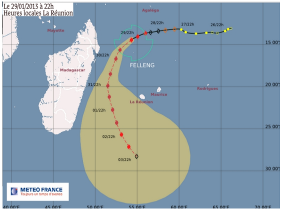
- Various applications require coupling an oceanic model and an atmospheric model.



climate modeling



seasonal forecasts



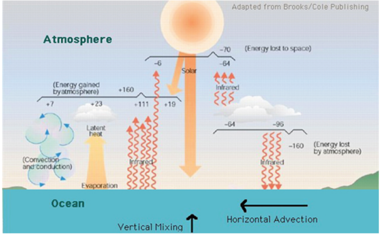
short term predictions

- **Objective:** revisit the coupling strategies presently used in such systems

Air-sea interactions

$$\begin{cases} \mathcal{L}_{\text{atm}}\, \boldsymbol{U}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}}\, \boldsymbol{U}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \end{cases}$$

$$\text{with } \boldsymbol{U}^{\text{a}} = \begin{pmatrix} \boldsymbol{u}_h^{\text{a}} \\ T^{\text{a}} \end{pmatrix} \quad \boldsymbol{U}^{\text{o}} = \begin{pmatrix} \boldsymbol{u}_h^{\text{o}} \\ T^{\text{o}} \end{pmatrix}$$

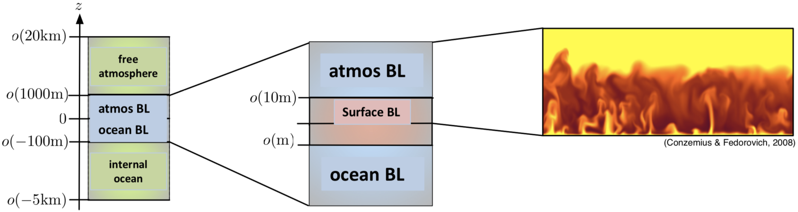


Interface conditions:

momentum $\quad \rho^{\text{a}} K_{\text{m}}^{\text{a}} \dfrac{\partial \boldsymbol{u}_h^{\text{a}}}{\partial z} = \rho^{\text{o}} K_{\text{m}}^{\text{o}} \dfrac{\partial \boldsymbol{u}_h^{\text{o}}}{\partial z} = \boldsymbol{\tau} \quad$ on $\Gamma \times [0, T]$

heat flux $\quad \rho^{\text{a}} c_p^{\text{a}} K_{\text{T}}^{\text{a}} \dfrac{\partial T^{\text{a}}}{\partial z} = \rho^{\text{o}} c_p^{\text{o}} K_{\text{T}}^{\text{o}} \dfrac{\partial T^{\text{o}}}{\partial z} = Q_{\text{S}} + \mathcal{R} \quad$ on $\Gamma \times [0, T]$

Estimation of OA fluxes



$$\begin{cases} \boldsymbol{\tau} = \rho^{\mathrm{a}}\, C_D \, \|[\boldsymbol{u}]_o^a\| \, [\boldsymbol{u}]_o^a \\ Q_S = \rho^{\mathrm{a}} c_p^{\mathrm{a}}\, C_H \, \|[\boldsymbol{u}]_o^a\| \, [T]_o^a \end{cases}$$

$[\boldsymbol{u}]_o^a, [T]_o^a =$ jumps of $\boldsymbol{u}$ and T

C_D, C_H: exchange coefficients given by (complicated) bulk formulas $= f([\boldsymbol{u}]_o^a, [T]_o^a, [q]_o^a, z_{\mathrm{atm}}, z_{\mathrm{oce}}, \ldots)$

Keywords: Monin-Obukhov theory (wall law + stratified fluid), bulk aerodynamic formulas...

Pelletier C., F. Lemarié, E. Blayo, J.-L. Redelsperger and P.-E. Brilouet, 2018: Comprehensive and coupled ocean-atmosphere turbulent parameterization schemes. *To be submitted*.

In summary

We have to solve:

$$\begin{cases} \mathcal{L}_{\text{atm}}\, \boldsymbol{U}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}}\, \boldsymbol{U}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \\ \mathcal{F}_{\text{atm}}\, \boldsymbol{U}^{\text{a}} = \mathcal{F}_{\text{oce}}\, \boldsymbol{U}^{\text{o}} = F_{\text{OA}}(\boldsymbol{U}^{\text{a}}, \boldsymbol{U}^{\text{o}}, \mathcal{R}) & \text{on } \Gamma \times [0, T] \end{cases}$$

In summary

We have to solve:

$$\begin{cases} \mathcal{L}_{\text{atm}}\, \boldsymbol{U}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}}\, \boldsymbol{U}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \\ \mathcal{F}_{\text{atm}}\, \boldsymbol{U}^{\text{a}} = \mathcal{F}_{\text{oce}}\, \boldsymbol{U}^{\text{o}} = F_{\text{OA}}(\boldsymbol{U}^{\text{a}}, \boldsymbol{U}^{\text{o}}, \mathcal{R}) & \text{on } \Gamma \times [0, T] \end{cases}$$

Two actual approaches:

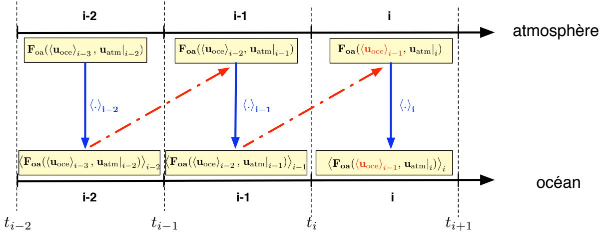
- Asynchronous coupling by time windows (averaged fluxes)
- Synchronous coupling at the time step (instantaneous fluxes)

Asynchronous coupling (time windows: $[0, T] = \cup_{i=1}^{M} [t_i, t_{i+1}]$)

$$\begin{cases} \mathcal{L}_{\mathrm{atm}} \boldsymbol{U}^{\mathrm{a}} = f_{\mathrm{atm}} & \text{in } \Omega_{\mathrm{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\mathrm{atm}} \boldsymbol{U}^{\mathrm{a}} = F_{\mathrm{OA}}(\langle \boldsymbol{U}^{\mathrm{o}} \rangle_{i-1}, \boldsymbol{U}^{\mathrm{a}}, \mathcal{R}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

then

$$\begin{cases} \mathcal{L}_{\mathrm{oce}} \boldsymbol{U}^{\mathrm{o}} = f_{\mathrm{oce}} & \text{in } \Omega_{\mathrm{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\mathrm{oce}} \boldsymbol{U}^{\mathrm{o}} = \langle \mathcal{F}_{\mathrm{atm}} \boldsymbol{U}^{\mathrm{a}} \rangle_i & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$



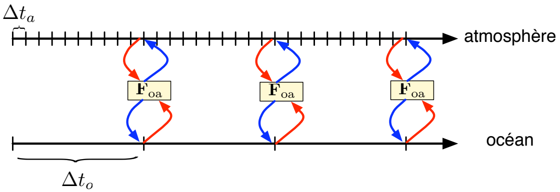
- same mean fluxes over each time window $[t_i, t_{i+1}]$
- ... but synchrony issue

Synchronous coupling at the time step

$$\begin{cases} \mathcal{L}_{\text{atm}} \boldsymbol{U}^{\text{a}} & = & f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_i + N\,\Delta t_{\text{a}}] \\ \mathcal{F}_{\text{atm}} \boldsymbol{U}^{\text{a}} & = & F_{\text{OA}}(\color{red}{\boldsymbol{U}^{\text{o}}(t_i)}, \boldsymbol{U}^{\text{a}}(t), \mathcal{R}(t)) & \text{on } \Gamma \times [t_i, t_i + N\,\Delta t_{\text{a}}] \end{cases}$$

and

$$\begin{cases} \mathcal{L}_{\text{oce}} \boldsymbol{U}^{\text{o}} & = & f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_i + \Delta t_{\text{o}}] \\ \mathcal{F}_{\text{oce}} \boldsymbol{U}^{\text{o}} & = & F_{\text{OA}}(\color{red}{\boldsymbol{U}^{\text{o}}(t_i)}, \color{red}{\boldsymbol{U}^{\text{a}}(t_i)}, \color{red}{\mathcal{R}(t_i)}) & \text{on } \Gamma \times [t_i, t_i + \Delta t_{\text{o}}] \end{cases}$$

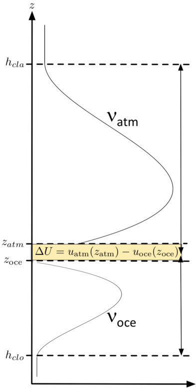


- still a (much smaller) synchrony issue
- difficult to implement efficiently
- validity issue

Stability analysis

Model problem: 1-D diffusion with variable and discontinuous coefficients

$$\begin{array}{rcll} \dfrac{\partial u_{\text{atm}}}{\partial t} - \dfrac{\partial}{\partial z}\left(\nu_{\text{atm}}(z)\dfrac{\partial u_{\text{atm}}}{\partial z}\right) & = & f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0,T] \\ \dfrac{\partial u_{\text{oce}}}{\partial t} - \dfrac{\partial}{\partial z}\left(\nu_{\text{oce}}(z)\dfrac{\partial u_{\text{oce}}}{\partial z}\right) & = & f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0,T] \end{array}$$



with interface conditions:

Dirichlet-Neumann: $\left\{ \begin{array}{c} u_{\text{atm}}(0,t) = u_{\text{oce}}(0,t) \\ \nu_{\text{oce}}(0)\dfrac{\partial u_{\text{oce}}}{\partial z}(0,t) = \nu_{\text{atm}}(0)\dfrac{\partial u_{\text{atm}}}{\partial z}(0,t) \end{array} \right.$

or

linearized bulk:

$$\nu_{\text{atm}}(0)\frac{\partial u_{\text{atm}}}{\partial z}(0,t) = \nu_{\text{oce}}(0)\frac{\partial u_{\text{oce}}}{\partial z}(0,t) = \alpha\left(u_{\text{atm}}(0^+,t) - u_{\text{oce}}(0^-,t)\right)$$

Stability analysis: synchronous coupling

Consider a natural discretization scheme: Euler + implicit $\longrightarrow$ each model is unconditionally stable.

- Stability analysis shows that the coupled scheme is unstable for usual configurations of ocean-atmosphere models

$$
\text{stable iff} \quad \begin{cases} \Delta z_{\text{atm}} \ll \Delta z_{\text{oce}} & \text{for Dirichlet-Neumann} \\ \alpha \ \leq \min \left(\dfrac{\nu_{\text{oce}}(0)}{\Delta z_{\text{oce}}} \dfrac{\rho_{\text{oce}}}{\rho_{\text{atm}}} \, ; \, \dfrac{\nu_{\text{atm}}(0)}{\Delta z_{\text{atm}}} \right) & \text{for linearized bulk} \end{cases}
$$

Lemarié F., E. Blayo and L. Debreu, 2015: Analysis of ocean-atmosphere coupling algorithms: consistency and stability issues. *Procedia Computer Science*, **51**, 2066-2075.

- This does not mean that realistic coupled OA models systematically blow up, due to other processes (additional diffusion and viscosity).

Beljaars A., E. Dutra, G. Balsamo and F. Lemarié, 2017: On the numerical stability of surface–atmosphere coupling in weather and climate models. *Geosci. Model Dev.*, **10**, 977–989.

Conclusion on current coupling methods

- Current coupling methods are simple ad-hoc algorithms, which ensure that fluxes are balanced and are computationally cheap.
- These methods are inadequate from a mathematical point of view.

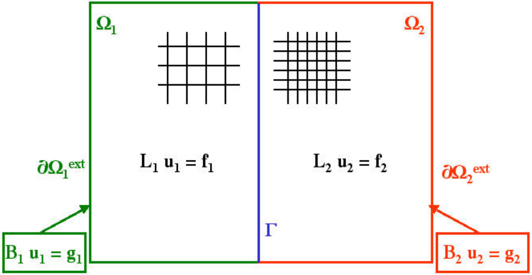
Conclusion on current coupling methods

- Current coupling methods are simple ad-hoc algorithms, which ensure that fluxes are balanced and are computationally cheap.
- These methods are inadequate from a mathematical point of view.

Issues:

- Can we improve the coupling method ?
- Does it improve the physics of the coupled solution ?
- Can this be done for a reasonable CPU cost ?

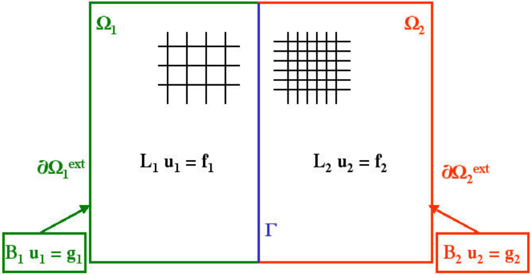
A convenient framework: Schwarz methods



$$\left\{\begin{array}{lll} L_1 u_1 & = f_1 & \Omega_1 \times [0,T] \\ u_1 & \text{given} & \text{at } t=0 \\ B_1 u_1 & = g_1 & \partial\Omega_1^{ext} \times [0,T] \end{array}\right. \qquad \left\{\begin{array}{lll} L_2 u_2 & = f_2 & \Omega_2 \times [0,T] \\ u_2 & \text{given} & \text{at } t=0 \\ B_2 u_2 & = g_2 & \partial\Omega_2^{ext} \times [0,T] \end{array}\right.$$

$+$ physical constraints at the interface : $\mathcal{F}(u_1, u_2) = 0$

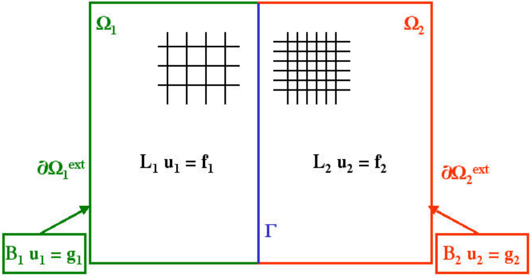
A convenient framework: Schwarz methods



$$\left\{\begin{array}{rll} L_1 u_1 & = f_1 & \Omega_1 \times [0,T] \\ u_1 & \text{given} & \text{at } t=0 \\ B_1 u_1 & = g_1 & \partial\Omega_1^{ext} \times [0,T] \\ C u_1 & = C' u_2 & \Gamma \times [0,T] \end{array}\right. \qquad \left\{\begin{array}{rll} L_2 u_2 & = f_2 & \Omega_2 \times [0,T] \\ u_2 & \text{given} & \text{at } t=0 \\ B_2 u_2 & = g_2 & \partial\Omega_2^{ext} \times [0,T] \\ D u_2 & = D' u_1 & \Gamma \times [0,T] \end{array}\right.$$

with $(Cu_1 = C'u_2 \;\text{ and }\; Du_2 = D'u_1) \iff \mathcal{F}(u_1, u_2) = 0$

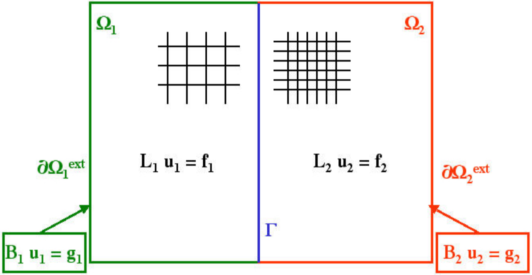
A convenient framework: Schwarz methods



$$\left\{\begin{array}{lll} L_1 u_1^{k+1} & = f_1 & \Omega_1 \times [0,T] \\ u_1^{k+1} & \text{given} & \text{at } t=0 \\ B_1 u_1^{k+1} & = g_1 & \partial\Omega_1^{ext} \times [0,T] \\ C u_1^{k+1} & = C' u_2^k & \Gamma \times [0,T] \end{array}\right. \qquad \left\{\begin{array}{lll} L_2 u_2^{k+1} & = f_2 & \Omega_2 \times [0,T] \\ u_2^{k+1} & \text{given} & \text{at } t=0 \\ B_2 u_2^{k+1} & = g_2 & \partial\Omega_2^{ext} \times [0,T] \\ D u_2^{k+1} & = D' u_1^k & \Gamma \times [0,T] \end{array}\right.$$

$$\text{with } (Cu_1 = C'u_2 \ \text{ and } Du_2 = D'u_1) \iff \mathcal{F}(u_1,u_2) = 0$$

A convenient framework: Schwarz methods



$$\left\{\begin{array}{rll} L_1 u_1^{k+1} & = f_1 & \Omega_1 \times [0,T] \\ u_1^{k+1} & \text{given} & \text{at } t=0 \\ B_1 u_1^{k+1} & = g_1 & \partial\Omega_1^{ext} \times [0,T] \\ C u_1^{k+1} & = C' u_2^k & \Gamma \times [0,T] \end{array}\right. \qquad \left\{\begin{array}{rll} L_2 u_2^{k+1} & = f_2 & \Omega_2 \times [0,T] \\ u_2^{k+1} & \text{given} & \text{at } t=0 \\ B_2 u_2^{k+1} & = g_2 & \partial\Omega_2^{ext} \times [0,T] \\ D u_2^{k+1} & = D' u_1^k & \Gamma \times [0,T] \end{array}\right.$$

$$\text{with } (Cu_1 = C'u_2 \;\text{ and }\; Du_2 = D'u_1) \iff \mathcal{F}(u_1,u_2) = 0$$

- Present ocean-atmosphere coupling methods correspond to one single iteration of a Schwarz-like coupling method

Toward iterative ocean-atmosphere coupling

$$\begin{cases} \mathcal{L}_{\text{atm}} \boldsymbol{U}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \boldsymbol{U}^{\text{a}} = F_{\text{OA}}(\boldsymbol{U}^{\text{o}}, \boldsymbol{U}^{\text{a}}, \mathcal{R}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

$$\begin{cases} \mathcal{L}_{\text{oce}} \boldsymbol{U}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \boldsymbol{U}^{\text{o}} = \mathcal{F}_{\text{atm}} \boldsymbol{U}^{\text{a}} & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

Toward iterative ocean-atmosphere coupling

Iterate until convergence

$$\begin{cases} \mathcal{L}_{\text{atm}} \boldsymbol{U}^{\text{a}}_{k+1} & = & f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \boldsymbol{U}^{\text{a}}_{k+1} & = & F_{\text{OA}}(\boldsymbol{U}^{\text{o}}_{k}, \boldsymbol{U}^{\text{a}}_{k+1}, \mathcal{R}_{k+1}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

then

$$\begin{cases} \mathcal{L}_{\text{oce}} \boldsymbol{U}^{\text{o}}_{k+1} & = & f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \boldsymbol{U}^{\text{o}}_{k+1} & = & \mathcal{F}_{\text{atm}} \boldsymbol{U}^{\text{a}}_{k+1} & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

Impact on the physics

A major difficulty There is no idealized coupled ocean-atmosphere testcase with a known reference solution.

Impact on the physics

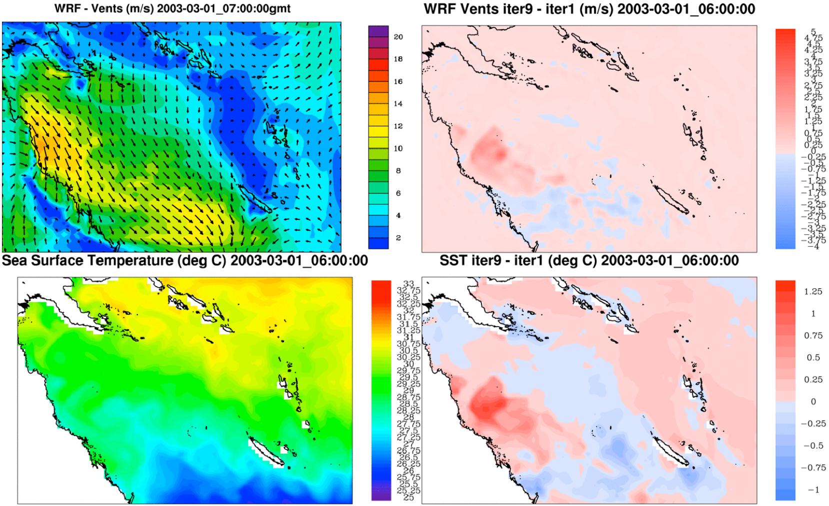
Simulation of the tropical cyclone Erica (2003), by coupling

- ROMS: primitive equation ocean model (Shchepetkin-McWilliams, 2005)
- WRF: non hydrostatic atmospheric model (Skamarock-Klemp, 2007)

$\Delta x_{\text{atm}} = 35\text{km}$, $\Delta t_{\text{atm}} = 180\text{s}$
$\Delta x_{\text{oce}} = 18\text{km}$, $\Delta t_{\text{oce}} = 1800\text{s}$
15-day simulation

Interface conditions: vertical fluxes for momentum, heat and fresh water

Impact on the physics (cont'd)



10-meter wind (m/s) and sea surface temperature (°C).

Impact on the physics (cont'd)

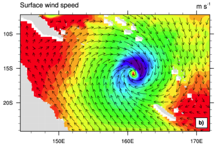
To assess the robustness of the coupled solution: ensemble simulations

- Initial conditions
- Length of the time windows: 6h vs 3h

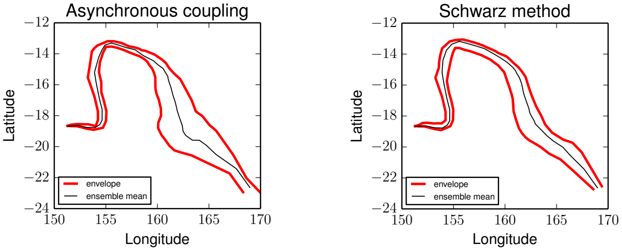
Impact on the physics (cont'd)

To assess the robustness of the coupled solution: ensemble simulations

- Initial conditions
- Length of the time windows: 6h vs 3h

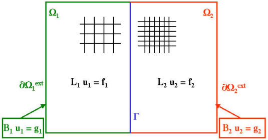


Trajectory of the cyclone



- The uncertainty on the cyclone trajectory and intensity is decreased by 30%-40%. (see Lemarié et al, 2014, for further details)

Decreasing the cost: absorbing boundary conditions

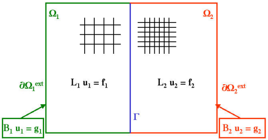


$$\left\{\begin{array}{rll} L_1 u_1^{k+1} & = f_1 & \Omega_1 \times [0,T] \\ u_1^{k+1} & \text{given} & \text{at } t=0 \\ B_1 u_1^{k+1} & = g_1 & \partial\Omega_1^{\text{ext}} \times [0,T] \\ C u_1^{k+1} & = C' u_2^k & \Gamma \times [0,T] \end{array}\right. \qquad \left\{\begin{array}{rll} L_2 u_2^{k+1} & = f_2 & \Omega_2 \times [0,T] \\ u_2^{k+1} & \text{given} & \text{at } t=0 \\ B_2 u_2^{k+1} & = g_2 & \partial\Omega_2^{\text{ext}} \times [0,T] \\ D u_2^{k+1} & = D' u_1^k & \Gamma \times [0,T] \end{array}\right.$$

Systems satisfied by the errors $e_i^k = u_i^k - u_i$:

$$\left\{\begin{array}{rll} L_1 e_1^{k+1} & = 0 & \Omega_1 \times [0,T] \\ e_1^{k+1} & = 0 & \text{at } t=0 \\ B_1 e_1^{k+1} & = 0 & \partial\Omega_1^{\text{ext}} \times [0,T] \\ C e_1^{k+1} & = C' e_2^k & \Gamma \times [0,T] \end{array}\right. \qquad \left\{\begin{array}{rll} L_2 e_2^{k+1} & = 0 & \Omega_2 \times [0,T] \\ e_2^{k+1} & = 0 & \text{at } t=0 \\ B_2 e_2^{k+1} & = 0 & \partial\Omega_2^{\text{ext}} \times [0,T] \\ D e_2^{k+1} & = D' e_1^k & \Gamma \times [0,T] \end{array}\right.$$

Decreasing the cost: absorbing boundary conditions



$$\begin{cases} L_1 u_1^{k+1} & = f_1 & \Omega_1 \times [0, T] \\ u_1^{k+1} & \text{given} & \text{at } t = 0 \\ B_1 u_1^{k+1} & = g_1 & \partial\Omega_1^{\text{ext}} \times [0, T] \\ C u_1^{k+1} & = C' u_2^k & \Gamma \times [0, T] \end{cases} \qquad \begin{cases} L_2 u_2^{k+1} & = f_2 & \Omega_2 \times [0, T] \\ u_2^{k+1} & \text{given} & \text{at } t = 0 \\ B_2 u_2^{k+1} & = g_2 & \partial\Omega_2^{\text{ext}} \times [0, T] \\ D u_2^{k+1} & = D' u_1^k & \Gamma \times [0, T] \end{cases}$$

Systems satisfied by the errors $e_i^k = u_i^k - u_i$:

$$\begin{cases} L_1 e_1^{k+1} & = 0 & \Omega_1 \times [0, T] \\ e_1^{k+1} & = 0 & \text{at } t = 0 \\ B_1 e_1^{k+1} & = 0 & \partial\Omega_1^{\text{ext}} \times [0, T] \\ C e_1^{k+1} & = C' e_2^k & \Gamma \times [0, T] \end{cases} \qquad \begin{cases} L_2 e_2^{k+1} & = 0 & \Omega_2 \times [0, T] \\ e_2^{k+1} & = 0 & \text{at } t = 0 \\ B_2 e_2^{k+1} & = 0 & \partial\Omega_2^{\text{ext}} \times [0, T] \\ D e_2^{k+1} & = D' e_1^k & \Gamma \times [0, T] \end{cases}$$

If one finds C', D' such that $C' e_2 = 0$ and/or $D' e_1 = 0$, then convergence in 2 iterations. $\longrightarrow$ **exact absorbing conditions** (Engquist & Majda, 77)

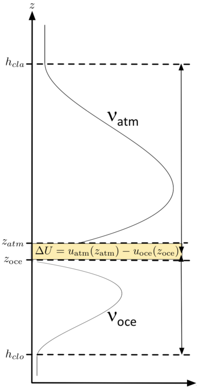
Improving the convergence speed

Model problem: coupling of two Ekman layers

$$\begin{aligned}
\partial_t u_{\text{atm}} - f\, v_{\text{atm}} - \partial_z \left(\nu_{\text{atm}}(z)\partial_z u_{\text{atm}}\right) &= F^x_{\text{atm}} & \\
\partial_t v_{\text{atm}} + f\, u_{\text{atm}} - \partial_z \left(\nu_{\text{atm}}(z)\partial_z v_{\text{atm}}\right) &= F^y_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\
u_{\text{atm}}(z, t=0) &= u_0(z) & z \in \Omega_{\text{atm}}
\end{aligned}$$

$$\begin{aligned}
\partial_t u_{\text{oce}} - f\, v_{\text{oce}} - \partial_z \left(\nu_{\text{oce}}(z)\partial_z u_{\text{oce}}\right) &= F^x_{\text{oce}} & \\
\partial_t v_{\text{oce}} + f\, u_{\text{oce}} - \partial_z \left(\nu_{\text{oce}}(z)\partial_z v_{\text{oce}}\right) &= F^y_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \\
u_{\text{oce}}(z, t=0) &= u_0(z) & z \in \Omega_{\text{oce}}
\end{aligned}$$

$$\begin{pmatrix} u_{\text{atm}} \\ v_{\text{atm}} \end{pmatrix} = \begin{pmatrix} u_{\text{oce}} \\ v_{\text{oce}} \end{pmatrix} \text{ and } \nu_{\text{atm}}(0)\,\partial_z \begin{pmatrix} u_{\text{atm}} \\ v_{\text{atm}} \end{pmatrix} = \nu_{\text{oce}}(0)\partial_z \begin{pmatrix} u_{\text{oce}} \\ v_{\text{oce}} \end{pmatrix} \quad \text{on } \Gamma \times [0, T]$$



Difficulties:

- coupling of u and v
- variable in space diffusivity + discontinuity at $z = 0$

Improving the convergence speed

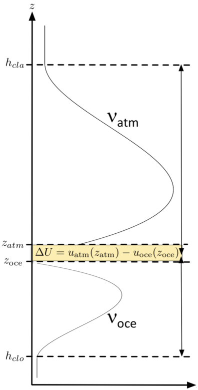
Model problem: coupling of two Ekman layers

Schwarz iteration:

$$
\begin{array}{rcll}
\partial_t u_{\text{atm}}^k - f\, v_{\text{atm}}^k - \partial_z \left(\nu_{\text{atm}}(z)\partial_z u_{\text{atm}}^k\right) & = & F_{\text{atm}}^x & \\
\partial_t v_{\text{atm}}^k + f\, u_{\text{atm}}^k - \partial_z \left(\nu_{\text{atm}}(z)\partial_z v_{\text{atm}}^k\right) & = & F_{\text{atm}}^y & \text{in } \Omega_{\text{atm}} \times \\
u_{\text{atm}}^k(z, t=0) & = & u_0(z) & z \in \Omega_{\text{atm}} \\
\mathcal{C}\, u_{\text{atm}}^k(0,t) & = & \mathcal{C}'\, u_{\text{atm}}^{k-1}(0,t) & t \in [0,T]
\end{array}
$$

$$
\begin{array}{rcll}
\partial_t u_{\text{oce}}^k - f\, v_{\text{oce}}^k - \partial_z \left(\nu_{\text{oce}}(z)\partial_z u_{\text{oce}}^k\right) & = & F_{\text{oce}}^x & \\
\partial_t v_{\text{oce}}^k + f\, u_{\text{oce}}^k - \partial_z \left(\nu_{\text{oce}}(z)\partial_z v_{\text{oce}}^k\right) & = & F_{\text{oce}}^y & \text{in } \Omega_{\text{oce}} \times [0, \\
u_{\text{oce}}^k(z, t=0) & = & u_0(z) & z \in \Omega_{\text{oce}} \\
\mathcal{D}\, u_{\text{oce}}^k(0,t) & = & \mathcal{D}'\, u_{\text{oce}}^{k}(0,t) & t \in [0,T]
\end{array}
$$

for a given $(u_{\text{oce}}^0, v_{\text{oce}}^0)$ on Γ.



Improving the convergence speed (cont'd)

		Constant diffusion	Space dependent diffusion
Steady state	No Coriolis effect	**Dubois (2007)** opt. Schwarz 2-D adv-diff eq. **Gander-Zhang (2016)** opt. Schwarz Helmholtz eq.	**Lions (1990)** convergence Schwarz diffusion eq.
	Coriolis effect		
Time dependent	No Coriolis effect	**Gander-Halpern (2002)** opt. Schwarz heat eq. **Blayo-Rousseau-Tayachi (2017)** lin. viscous SW eq. **Bennequin-Gander-Gouarin-Halpern (2004)** opt. Schwarz 2-D diff-reaction	**Lemarié-Debreu-Blayo (2013)** opt. Schwarz 1-D diffusion
	Coriolis effect	**Martin (2003)** opt. Schwarz 2-D SW **Audusse-Dreyfuss-Merlet (2010)** opt. Schwarz 3-D primitive eqs	

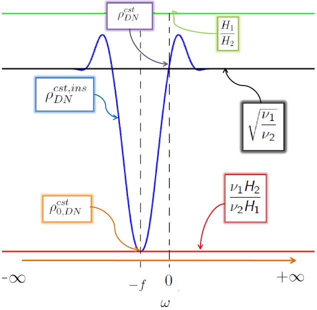
Improving the convergence speed (cont'd)

		Constant diffusion	Space dependent diffusion
Steady state	No Coriolis effect	**Dubois (2007)** opt. Schwarz 2-D adv-diff eq. **Gander-Zhang (2016)** opt. Schwarz Helmholtz eq.	**Lions (1990)** convergence Schwarz diffusion eq.
	Coriolis effect		
Time dependent	No Coriolis effect	**Gander-Halpern (2002)** opt. Schwarz heat eq. **Blayo-Rousseau-Tayachi (2017)** lin. viscous SW eq. **Bennequin-Gander-Gouarin-Halpern (2004)** opt. Schwarz 2-D diff-reaction	**Lemarié-Debreu-Blayo (2013)** opt. Schwarz 1-D diffusion
	Coriolis effect	**Martin (2003)** opt. Schwarz 2-D SW **Audusse-Dreyfuss-Merlet (2010)** opt. Schwarz 3-D primitive eqs	

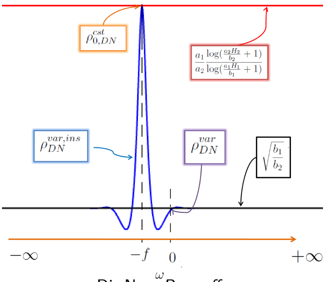
Improving the convergence speed (cont'd)

Exact analytical expression for the convergence factor:

- P_0, P_1 and P_2 diffusion profiles
- Dirichlet-Neumann and Robin-Robin interface conditions
- optimized coefficients for Robin-Robin conditions



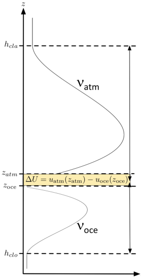
Dir-Neu, cst coeff



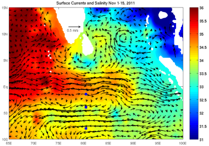
Dir-Neu, P_1 coeff

Blayo E., F. Lemarié, C. Pelletier and S. Thery, 2018: Coupling two Ekman layers with a Schwarz algorithm. *In preparation*.

Current and future work



- Integrate all these results in a model of the three boundary layers (atmospheric, surface, oceanic)
- Study the coherence of the different parameterizations (well-posedness issues)



In collaboration with climate scientists:

- Build 1D coupled reference test cases (idealized and realistic)
- Include this coupling strategy in the French IPSL climate model
- Mitigate the cost (perform the iterations only for the boundary layers, model reduction...)